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J. Phys. A: Math. Gen. 25 (1992) 5463–5473. Printed in the UK

CAM approach to the ground-state phase transition in the two-dimensional transverse Ising model

Yoshihiko Nonomura and Masuo Suzuki
Department of Physics, Faculty of Science, University of Tokyo, Hongo, Bunkyo-ku, Tokyo 113, Japan

Received 30 March 1992

Abstract. The double-cluster approximation (DCA) and the multi-effective-field approximation (MEFA) are formulated for the two-dimensional $S = \frac{1}{2}$ transverse Ising model on a square lattice in the ground state. These two cluster-effective-field approximations expand the sizes of clusters effectively. Combining some suitable series of these approximations with the coherent-anomaly method (CAM), we have analysed the critical phenomenon of this model. We have obtained $g_c^* = 1.528 \pm 0.016$ and $\gamma = 1.25 \pm 0.08$ using the DCA, and $\gamma = 1.26 \pm 0.10$ using the MEFA. These estimates are consistent with the other results, $g_c^* \simeq 1.52$ and $\gamma \simeq 1.24$. Our scheme is expected to be useful in the study of frustrated systems, because various shapes of clusters can be used in our calculations.

1. Introduction

Recently the ground-state properties of two-dimensional frustrated quantum spin systems have been intensively studied, because their properties seem to be in close relation with the mechanism of high-temperature superconductors.

Although numerous studies of these problems have already been done, even their ground-state properties are not yet clearly understood. The spin–wave (or modified spin–wave [1]) calculation [2–5] is nothing but an approximation, only small systems can be treated in the exact-diagonalization calculation [3, 6–8], and the negative sign is very severe in quantum Monte Carlo calculation.

In the present paper we propose a new approach to ground-state phase transitions in two-dimensional quantum spin systems. It is based on cluster-effective-field approximations and the coherent-anomaly method (CAM) [9–11]. Although the sizes of the available clusters are of the same order as the ones used in the exact-diagonalization calculation, their sizes are *effectively* expanded in our new scheme. Each approximation shows a classical singularity in the vicinity of its mean-field critical point, true critical phenomena can be obtained from *a series of* approximations using the CAM. Conversely, we can judge whether a series of approximations is good or not by their coherent anomaly.

The purpose of this paper is a test of our scheme before applying it to quantum antiferromagnets and frustrated quantum spin systems. We use the double-cluster approximation (DCA) [12–18] and the multi-effective-field approximation (MEFA) [15, 19–22] for the two-dimensional $S = \frac{1}{2}$ transverse Ising model on a square lattice:

$$\mathcal{H} = -\sum_{\langle ij \rangle} S_i^z S_j^z - g \sum_i S_i^x - H \sum_i S_i^z \tag{1.1}$$

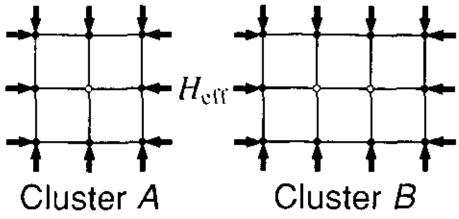

Figure 1. The way of application of the effective field H_{eff} in the DCA. The 3×3 and 4×3 clusters are used here.

where $\sum_{\langle ij \rangle}$ denotes the sum over all the nearest-neighbour bonds. Since the critical phenomenon of this model have already been extensively studied [23–27], our estimates based on the CAM can be compared with these previous results.

In section 2, the DCA is formulated in the present model. The critical point g_c^* and the critical exponent γ are estimated using the DCA and the CAM. In section 3, we present a similar formulation and estimation based on the MEFA and the CAM. Section 4 is devoted to a summary and discussion of these descriptions.

2. CAM analysis of the DCA series

In the present section we formulate the DCA for the two-dimensional $S = \frac{1}{2}$ transverse Ising model on a square lattice. Both an equation to determine the mean-field critical point and an expression for the critical coefficient of the zero-field magnetic susceptibility are obtained. A brief review of the CAM is also given in this section. The values of the critical point and the critical exponent are estimated from this series of approximations using the CAM.

2.1. Formulation of the DCA

In this approximation we consider *two different* clusters A and B, and apply the *same* effective field H_{eff} on their boundary spins (figure 1). The effective Hamiltonian of the N_I-spin cluster Ω_I ($I = A$ or B) is given by

$$\mathcal{H}_{N_I} = -\sum_{\langle ij \rangle \in \Omega_I} S_i^z S_j^z - g\sum_{i=1}^{N_I} S_i^x - H\sum_{i=1}^{N_I} S_i^z - H_{\text{eff}}\sum_{j=1}^{N_I} z_j S_j^z \qquad (2.1)$$

where z_j denotes the number of effective-field bonds on the site j. Namely, on a square lattice,

$$z_j = \begin{cases} 2 & \text{on a corner} \\ 1 & \text{on an edge} \\ 0 & \text{in the bulk.} \end{cases} \qquad (2.2)$$

The required self-consistency condition is

$$\frac{1}{N_A}\sum_{i=1}^{N_A}\langle S_i^z \rangle_g^A = \frac{1}{N_B}\sum_{i=1}^{N_B}\langle S_i^z \rangle_g^B \qquad (2.3)$$

where $\langle \cdots \rangle_g^I$ denotes the ground-state average in the cluster I.

As was explained in the previous paper [17], the order parameter of the cluster I can be expanded with respect to H_{eff} and H:

$$\frac{1}{N_I}\sum_{i=1}^{N_I}\langle S_i^z\rangle_{\mathrm{g}}^I = H_{\mathrm{eff}}\frac{1}{N_I}\sum_{i=1}^{N_I}\frac{\partial}{\partial H_{\mathrm{eff}}}\langle S_i^z\rangle_{\mathrm{g}}^I\bigg|_{H_{\mathrm{eff}}=0} + H\frac{1}{N_I}\sum_{i=1}^{N_I}\frac{\partial}{\partial H}\langle S_i^z\rangle_{\mathrm{g}}^I\bigg|_{H=0} + \cdots \tag{2.4}$$

$$\equiv P^I H_{\mathrm{eff}} + R^I H + \cdots . \tag{2.5}$$

The mean-field critical point g_{c} is determined [17] as the solution of the following equation

$$P^A = P^B \tag{2.6}$$

and the susceptibility at $g = g_{\mathrm{c}} + 0$ is given [17] by

$$\chi^I = \bar{\chi}_+^I\left(\frac{g_{\mathrm{c}}}{g - g_{\mathrm{c}}}\right)^{\gamma_{\mathrm{cl}}} \qquad \gamma_{\mathrm{cl}} = 1 \tag{2.7}$$

$$\bar{\chi}_+^I = -\frac{1}{g_{\mathrm{c}}}\frac{P^I(R^A - R^B)}{\frac{\mathrm{d}}{\mathrm{d}g}(P^A - P^B)\Big|_{g=g_{\mathrm{c}}}} \tag{2.8}$$

where $\bar{\chi}_+^A = \bar{\chi}_+^B$ because of condition (2.6).

In the CAM, the critical coefficient $\bar{\chi}_+$ is scaled [9, 10] as

$$\bar{\chi}_+ \sim \frac{\text{constant}}{\delta(g_{\mathrm{c}})^{\psi}} \qquad \text{for } g_{\mathrm{c}} \to g_{\mathrm{c}}^* \tag{2.9}$$

in some suitable series of approximations (canonical series). Here the 'degree of approximation' $\delta(g_{\mathrm{c}})$ is defined by

$$\delta(g_{\mathrm{c}}) \equiv \frac{g_{\mathrm{c}} - g_{\mathrm{c}}^*}{g_{\mathrm{c}}^*} \tag{2.10}$$

and the true critical exponent γ is obtained [9, 10] from the following relation,

$$\gamma = \gamma_{\mathrm{cl}} + \psi. \tag{2.11}$$

All the quantities in the formulae (2.6) and (2.8) are expressed by derivatives of the order parameter with respect to H, H_{eff} or g (at most twice). Such derivatives can be calculated sufficiently precisely by numerical differentiation [28]. For example, the quantity P^I can be evaluated by

$$P^I = \frac{1}{N_I}\sum_{i=1}^{N_I}\langle S_i^z\rangle_{\mathrm{g}}^I\bigg|_{\Delta H_{\mathrm{eff}}}\bigg/\Delta H_{\mathrm{eff}} \tag{2.12}$$

where ΔH_{eff} denotes the difference of H_{eff} from its critical value, zero. Since only the ground state of the cluster Hamiltonian is used in this calculation, we can utilize the Lanczos algorithm and treat large (here, up to 5×4) clusters.

2.2. Estimation of the non-classical critical exponent

According to applications of the DCA to the two-dimensional Ising model [16] and the two-dimensional $\pm J$ Ising spin glass [18], the shape of each cluster should be similar, namely it should be a square or a near-square rectangle. In the present paper we have treated 2-spin (2×1), 4-spin (2×2), 6-spin (3×2), 9-spin (3×3), 12-spin (4×3), 16-spin (4×4) and 20-spin (5×4) clusters.

Since numerical differentiation is used in the present calculation, we should take care of the accuracy of the results. All the excited states of the cluster Hamiltonian can be evaluated easily in small (here, up to 9-spin) clusters, and the calculation without numerical differentiation [17] is possible. Comparing the results obtained from these two calculation, we have optimized the differences $\Delta H, \Delta H_{\rm eff}$ and Δg. Finally, the values given by these two methods come to be consistent with one another up to the order $\sim 10^{-10}$ in g_c, and $\sim 10^{-4}$ in $\bar{\chi}_+$.

Results for various pairs of clusters are given in table 1. Generally speaking, approximations obtained from smaller clusters do not show good scaling properties. Thus, we have made the least-squares fitting for the 6–12, 9–12, ..., 16–20 approximations. We have assumed the simple CAM scaling form (2.9) and obtained (see figure 2)

$$g_c^* = 1.528 \pm 0.016 \qquad \gamma = 1.25 \pm 0.08. \tag{2.13}$$

Here the errors indicate the standard deviations in the estimates obtained by the fitting. The deviation of $\bar{\chi}_+$ is of the order $\sim 10^{-3}$ in each approximation. It is much larger than the error which originates from numerical differentiation. Thus, in the present estimation, we have neglected the error coming from numerical differentiation.

Table 1. The mean-field critical point g_c and the critical coefficient $\bar{\chi}_+$ in the DCA.

N_A	N_B	g_c	$\bar{\chi}_+$
2	4	1.625 646	0.4034
2	6	1.618 988	0.4122
4	6	1.609 027	0.4259
4	9	1.597 069	0.4442
6	9	1.589 349	0.4569
6	12	1.586 156	0.4642
9	12	1.581 846	0.4742
9	16	1.575 423	0.4890
12	16	1.570 844	0.5000
12	20	1.568 950	0.5063
16	20	1.566 547	0.5145

3. CAM analysis of the MEFA series

In the present section we formulate the MEFA for the two-dimensional $S = \frac{1}{2}$ transverse Ising model on a square lattice. Both simultaneous equations to determine the mean-field critical point together with an expression for the critical coefficient of the zero-field magnetic susceptibility are obtained. The values of the critical point and the critical exponent are estimated from this series of approximations using the CAM.

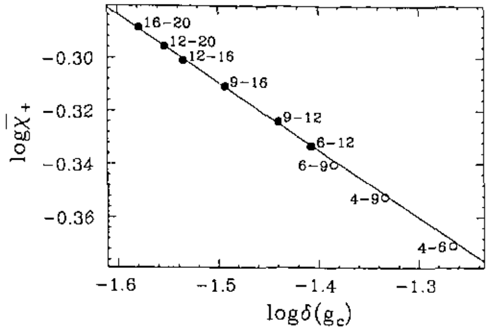

Figure 2. Coherent anomaly of the DCA series. $\operatorname{Log}\bar{\chi}_+$ is plotted against $\log\delta(g_c)$. Each pair of numbers denotes N_A and N_B. The straight line, the slope of which is equal to $-\psi$, is determined by the least-squares fitting using the data represented by the full circles.

3.1. Formulation of the MEFA

In this approximation we consider not only one-body effective fields but also *multi-body* effective fields. Even if we fix the size of the cluster, we can obtain various approximations by altering the combination of multi-body effective fields. The effective Hamiltonian of the cluster Ω is given by

$$\mathcal{H}^{\text{eff}} = -\sum_{\langle ij\rangle\in\Omega} S_i^z S_j^z - g\sum_{i=1}^{N} S_i^x - H\sum_{i=1}^{N} S_i^z - \sum_j J_j Q_j^{\text{even}} - \sum_j H_j Q_j^{\text{odd}} \tag{3.1}$$

where the quantity Q_j^{even} (Q_j^{odd}) denotes the sum of even (odd) number products of spins on which the corresponding even (odd) effective field J_j (H_j) is applied.

For example, on the 3×3 cluster (figure 3), explicit expressions for $\{Q_j\}$ are given as follows

$$\text{Odd}\begin{cases} Q_1^{\text{odd}} = S_1^z + S_3^z + S_5^z + S_7^z \\ Q_2^{\text{odd}} = S_2^z + S_4^z + S_6^z + S_8^z \\ \vdots \end{cases} \tag{3.2}$$

$$\text{Even}\begin{cases} Q_1^{\text{even}} = S_1^z S_2^z + S_2^z S_3^z + S_3^z S_4^z + S_4^z S_5^z + S_5^z S_6^z + S_6^z S_7^z + S_7^z S_8^z + S_8^z S_1^z \\ Q_2^{\text{even}} = S_1^z S_3^z + S_3^z S_5^z + S_5^z S_7^z + S_7^z S_1^z \\ \vdots \end{cases} \tag{3.3}$$

and the required self-consistency conditions are

$$\text{Odd}\begin{cases} \langle\Delta Q_1^{\text{odd}}\rangle_{\text{g}} \equiv \langle S_1^z\rangle_{\text{g}} - \langle S_0^z\rangle_{\text{g}} = 0 \\ \langle\Delta Q_2^{\text{odd}}\rangle_{\text{g}} \equiv \langle S_2^z\rangle_{\text{g}} - \langle S_0^z\rangle_{\text{g}} = 0 \\ \vdots \end{cases} \tag{3.4}$$

$$\text{Even}\begin{cases} \langle\Delta Q_1^{\text{even}}\rangle_{\text{g}} \equiv \langle S_1^z S_2^z\rangle_{\text{g}} - \langle S_8^z S_0^z\rangle_{\text{g}} = 0 \\ \langle\Delta Q_2^{\text{even}}\rangle_{\text{g}} \equiv \langle S_1^z S_3^z\rangle_{\text{g}} - \langle S_8^z S_4^z\rangle_{\text{g}} = 0 \\ \vdots \end{cases} \tag{3.5}$$

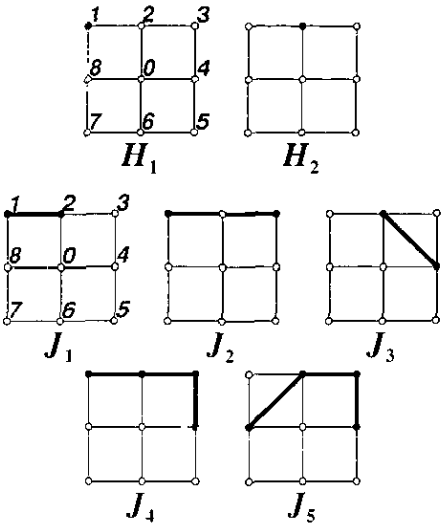

Figure 3. The multi-body effective fields applied on the 3 x 3 cluster.

The key point of the present formulation is that the odd effective fields $\{H_j\}$ vanish above the mean-field critical point g_c, but that the even effective fields $\{J_j\}$ always take non-vanishing values. Then the ground-state average of a quantity Q can be written as

$$\langle Q\rangle_g = \langle Q\rangle_g^* + \sum_j H_j \left.\frac{\partial}{\partial H_j}\langle Q\rangle_g^*\right|_{H_j=0} + H \left.\frac{\partial}{\partial H}\langle Q\rangle_g^*\right|_{H=0} + \cdots \tag{3.6}$$

where $\langle\cdots\rangle_g^*$ denotes the ground-state average with respect to the following effective Hamiltonian,

$$\mathcal{H}^* \equiv -\sum_{\langle ij\rangle\in\Omega} S_i^z S_j^z - g\sum_{i=1}^{N} S_i^x - \sum_j J_j Q_j^{\text{even}}. \tag{3.7}$$

The mean-field critical point g_c is given [20] by the solution of the following equation,

$$\det M = 0 \qquad (M)_{ij} \equiv \left.\frac{\partial}{\partial H_j}\langle\Delta Q_i\rangle_g^*\right|_{H_j=0}. \tag{3.8}$$

The unknown parameters $\{J_j\}$ are included in the explicit expression of (3.8). Thus, the values of g_c and $\{J_j\}$ are determined as the solutions of the simultaneous equations which consist of (3.5) and (3.8). The susceptibility at $g = g_c + 0$ is given by [22]

$$\chi = \bar{\chi}_+ \left(\frac{g_c}{g-g_c}\right)^{\gamma_{cl}} \qquad \gamma_{cl} = 1 \tag{3.9}$$

$$\bar{\chi}_+ = -\frac{1}{g_c}\sum_{i,j} \left.\frac{\partial}{\partial H}\langle\Delta Q_i^{\text{odd}}\rangle_g^*\right|_{H=0} \tilde{M}_{ij} \left.\frac{\partial}{\partial H_j}\langle S_0^z\rangle_g^*\right|_{H_j=0} \Big/ \left.\frac{\mathrm{d}}{\mathrm{d}g}(\det M)\right|_{g=g_c} \tag{3.10}$$

where $\tilde{M}_{ij}$ denotes the cofactor of the matrix M, and S_0^z means the centre spin of the cluster just as in the 3×3 cluster.

3.2. Estimation of the non-classical critical exponent

Here we have treated the 3×3 and 4×4 clusters. All the effective fields used here are given in figures 3 and 4, where only one typical spin product is displayed. Other spin products are obtained from its rotation and reflection. Although any combinations of effective fields are possible in principle, our previous studies [20, 22] showed that only some limited series of approximations can be used as canonical series. In such series of approximations the effective fields are applied almost in order of their strength, from the larger to the smaller. Thus, in order to see their strength, we have first applied all the effective fields given in figures 3 and 4. Then we have determined suitable combinations of the effective fields.

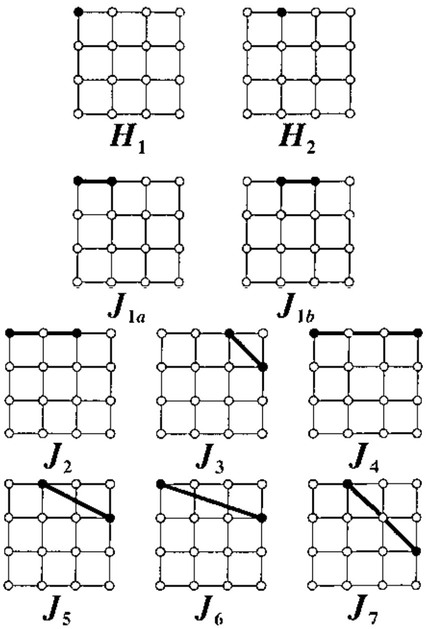

Figure 4. The multi-body effective fields applied on the 4×4 cluster.

Results obtained from the 3×3 and 4×4 clusters are given in tables 2 and 3. These data are plotted in figure 5, where we have used the estimate $g_c^* = 1.528$ obtained from the DCA.

These results show that approximations are improved when we consider multi-body effective fields: the mean-field critical points obtained from the 3×3 cluster plus multi-body effective fields are lower than the one obtained from the 4×4 cluster plus only one-body effective fields (the Bethe-like approximation [10]). The ones obtained from the 4×4 cluster plus multi-body effective fields are much lower.

Next, we see the coherent anomaly of these data. In the 3×3 cluster, the mean-field critical points hardly vary when the number of the multi-body effective fields is increased. Moreover, if we use $g_c^* = 1.528$, we have $\gamma \simeq 1.37$. This estimate is clearly different from the one obtained from the DCA. In the 4×4 cluster, the situation seems to be better. The mean-field critical points vary to some extent in accordance with altering the combination of effective fields, and we have $\gamma \simeq 1.29$ with $g_c^* = 1.528$. However, when the number of effective fields is increased, the data points tend to rapidly leave the CAM scaling line (see figure 5). Then it is still

Table 2. The mean-field critical point g_c, the critical coefficient $\bar{\chi}_+$ and the multi-body effective fields $\{J_j\}$ in the MEFA using the 3×3 cluster. Each approximation includes all the one-body effective fields and the even effective fields given in the bracket.

Approximations	g_c	$\bar{\chi}_+$	J_1	J_2	J_3	J_4	J_5
( )	1.602 038	0.4314	—	—	—	—	—
(1)	1.582 626	0.4751	0.123 22	—	—	—	—
(1,3)	1.583 876	0.4707	0.121 45	—	−0.003 77	—	—
(1,3,4)	1.583 869	0.4707	0.121 41	—	−0.003 77	0.000 41	—
(1,3,5)	1.585 419	0.4662	0.120 84	—	0.000 46	—	−0.037 13
(1,3,4,5)	1.585 405	0.4662	0.120 77	—	0.000 47	0.000 87	−0.037 23
(1,2)	1.581 958	0.4770	0.113 30	0.038 05	—	—	—
(1,2,4)	1.582 046	0.4766	0.113 73	0.037 98	—	−0.004 78	—
(1,2,5)	1.581 821	0.4775	0.113 31	0.038 44	—	—	0.001 61
(1,2,4,5)	1.581 892	0.4772	0.113 74	0.038 42	—	−0.004 84	0.001 83
(1,2,3)	1.581 441	0.4788	0.113 82	0.038 73	0.001 51	—	—
(1,2,3,4)	1.581 537	0.4783	0.114 23	0.038 65	0.001 49	−0.004 70	—
(1,2,3,5)	1.581 460	0.4787	0.113 83	0.038 67	0.001 54	—	−0.000 34
(1,2,3,4,5)	1.581 541	0.4783	0.114 23	0.038 64	0.001 49	−0.004 69	−0.000 07

difficult to make the least-squares fitting using only the data obtained from the 4×4 cluster. Thus, we use the (1) and $(1,2)$ approximations of the 3×3 cluster and the $(1a, 1b)$, $(1a, 1b, 2)$, $(1a, 1b, 3)$, $(1a, 1b, 2, 3)$ and $(1a, 1b, 2, 4)$ approximations of the 4×4 cluster, and we have

$$g_c^* = 1.542 \pm 0.006 \qquad \gamma = 1.17 \pm 0.04. \tag{3.11}$$

If use the value of g_c^* obtained from the DCA, the estimates are given by

$$g_c^* = 1.528 \pm 0.016 \qquad \gamma = 1.26 \pm 0.10. \tag{3.12}$$

4. Summary and discussion

In the present paper we have formulated the DCA and the MEFA for the two-dimensional $S = \frac{1}{2}$ transverse Ising model on a square lattice in the ground state, and estimated the critical point g_c^* and the critical exponent γ using these series of approximations and the CAM.

From the DCA, we have obtained

$$g_c^* = 1.528 \pm 0.016 \qquad \gamma = 1.25 \pm 0.08 \tag{4.1}$$

and from the MEFA (using the value of g_c^* estimated by the DCA),

$$g_c^* = 1.528 \pm 0.016 \qquad \gamma = 1.26 \pm 0.10. \tag{4.2}$$

Estimates obtained from other methods are given in table 4. These results are consistent with each other. Thus, it is confirmed that our methods are applicable to two-dimensional systems.

Then we inquire into the features of each approximation more precisely. In the DCA, the errors of the estimates seem to be large even though the coherent anomaly

Table 2. The mean-field critical point g_c, the critical coefficient $\bar{\chi}_+$ and the multi-body effective fields $\{J_j\}$ in the MEFA using the 4×4 cluster. Each approximation includes all the one-body effective fields and the even effective fields given in the bracket.

Approximations	g_c	$\bar{\chi}_+$	J_{1a}	J_{1b}	J_2	J_3	J_4	J_5	J_6	J_7
( )	1.585 089	0.4627	—	—	—	—	—	—	—	—
(1a,1b)	1.568 813	0.5104	0.138 84	0.138 55	—	—	—	—	—	—
(1a,1b,2)	1.566 829	0.5190	0.126 88	0.115 25	0.048 53	—	—	—	—	—
(1a,1b,3)	1.565 212	0.5229	0.141 74	0.139 36	—	0.034 60	—	—	—	—
(1a,1b,4)	1.568 114	0.5138	0.135 30	0.136 60	—	—	0.039 13	—	—	—
(1a,1b,3,4)	1.564 199	0.5304	0.137 70	0.137 09	—	0.036 20	0.045 22	—	—	—
(1a,1b,2,3)	1.562 611	0.5356	0.128 65	0.113 18	0.05405	0.037 33	—	—	—	—
(1a,1b,2,4)	1.566 682	0.5198	0.126 24	0.116 88	0.043 18	—	0.019 74	—	—	—
(1a,1b,2,5)	1.563 990	0.5325	0.130 16	0.113 15	0.051 79	—	—	0.013 72	—	—
(1a,1b,2,4,5)	1.563 710	0.5343	0.129 39	0.115 17	0.045 13	—	0.024 81	0.014 15	—	—
(1a,1b,2,3,4)	1.562 360	0.5371	0.127 84	0.115 22	0.047 45	0.037 89	0.024 31	—	—	—
(1a,1b,2,3,5)	1.562 021	0.5386	0.129 45	0.112 63	0.054 71	0.035 69	—	0.003 65	—	—
(1a,1b,2,3,6)	1.561 090	0.5455	0.129 27	0.113 69	0.055 04	0.036 32	—	—	0.013 17	—
(1a,1b,2,3,5,6)	1.560 888	0.5453	0.129 53	0.113 51	0.055 26	0.035 77	—	0.001 19	0.013 22	—
(1a,1b,2,3,4,5)	1.561 703	0.5407	0.128 65	0.114 74	0.047 81	0.03614	0.025 63	0.003 97	—	—
(1a,1b,2,3,4,6)	1.561 674	0.5414	0.128 06	0.115 51	0.047 72	0.037 48	0.024 90	—	0.005 89	—
(1a,1b,2,3,4,7)	1.562 685	0.5353	0.127 51	0.115 30	0.047 13	0.037 70	0.023 66	—	—	−0.002 81
(1a,1b,2,3,4,6,7)	1.561 992	0.5396	0.127 79	0.115 54	0.047 45	0.037 37	0.024 34	—	0.005 34	−0.002 16
(1a,1b,2,3,4,5,6)	1.561 207	0.5439	0.128 64	0.115 15	0.047 98	0.036 20	0.025 85	0.002 91	0.005 74	—
(1a,1b,2,3,4,5,7)	1.562 265	0.5374	0.128 09	0.114 89	0.047 26	0.035 81	0.024 50	0.003 97	—	−0.004 83
(1a,1b,2,3,4,5,6,7)	1.561 739	0.5407	0.128 20	0.115 20	0.047 52	0.035 93	0.024 92	0.003 10	0.004 75	−0.003 81

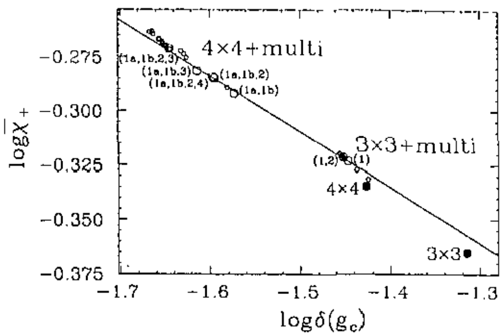

Figure 5. Coherent anomaly of the MEFA series (open circles) and the corresponding Bethe-like approximations (full circles). Log $\bar{\chi}_+$ is plotted against $\log\delta(g_c)$. The straight line, the slope of which is equal to $-\psi$, is determined by the least-squares fitting using the data represented by the large open circles and the value of g_c^* obtained from the DCA.

Table 4. The estimates of g_c^* and γ obtained from other methods. The high-temperature expansion method is abbreviated to HTE, the low-temperature expansion method to LTE and the finite-size scaling method to FSS.

References	Methods	g_c^*	γ
Hamer and Irving [24]	HTE	1.520 ± 0.005	1.257 ± 0.010
Hamer and Guttmann [25]	HTE	1.522 ± 0.002	1.245 ± 0.004
Marland [26]	LTE	1.522 ± 0.001	1.25 ± 0.02
Henkel [27]	FSS	1.524 ± 0.001	1.24 ± 0.02

holds well in a wide range of $\delta(g_c)$. This arises from the fact that the aberration of the data is not small. For example, when we see the 12–16, 12–20 and 16–20 approximations, we find that the tangent of the line which connects these three data points is larger than the one given by the least-squares fitting (see figure 2). The reason may be that these three approximations are made by enlarging the cluster in only one direction, therefore a crossover to the critical phenomenon of the one-dimensional $S = \frac{1}{2}$ transverse Ising model ($\gamma = 1.75$) appears. In short, the shapes of clusters are still important in the DCA, even though they do not explicitly appear in the calculation.

A similar argument is possible for the MEFA. As is well known, a two-dimensional quantum spin system in the ground state corresponds to a three-dimensional classical spin system. In the present calculation we have only applied the diagonal effective fields, which correspond to an effective enlargement of clusters only toward the real directions. Thus, the previous crossover may appear when the number of multi-body effective fields is increased. In fact, the data points become larger than the CAM scaling line when too many effective fields are applied on clusters.

Although this effect can be avoided by using the approximations including only small numbers of multi-body effective fields, it is still difficult to estimate the values of g_c^* and γ at the same time. In fact, in the present calculation, we have obtained insufficient estimates ($g_c^* = 1.542 \pm 0.006$, $\gamma = 1.17 \pm 0.04$). The reason is that the mean-field critical points do not vary so much even if the number of multi-body effective fields is increased. Then the present estimation is similar to a two-point (the data of the 3×3 cluster and the 4×4 cluster) fitting and therefore not accurate. This shortcoming will be reformed if we take other boundary conditions [29] or consider the off-diagonal multi-body effective fields and the effective fields applied onto the bulk of the clusters.

Acknowledgments

One of the present authors (YN) is grateful for financial support from the Japan Society for the Promotion of Science for Japanese Junior Scientists. This work was supported by the Grants-in-Aid for Scientific Research on Priority Areas 'Computational Physics as a New Frontier in Condensed Matter Research' and for Encouragement of Young Scientists, from the Ministry of Education, Science and Culture, Japan. Our computer programs are based on the Hamiltonian-diagonalization subroutine package 'TITPACK VER. 2' by H Nishimori. The numerical calculations were performed on the HITAC S820/80 of the Computer Centre, University of Tokyo and on the VAX6440 of the Meson Science Laboratory, Faculty of Science, University of Tokyo.

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